# QFT Spring2025

## HW1

### Q1 Black-body radiation

a) The energy spectrum of photons in the momentum space reads

$$u_E(k,T) \propto k^2 \frac{k}{e^{k/T}-1}.$$

Show that the maximum of $u_E(k,T)$ occurs at

$$(3-k/T)\, e^{k/T} - 3 = 0.$$

Solve the equation numerically and verify that $k/T \approx 2.8$. This is Wien's displacement Law.

b) Repeat the above exercise but instead work in the wavelength space.

Show that

$$u_\lambda(\lambda,T) \propto \frac{1}{\lambda^5} \frac{1}{e^{\frac{2\pi}{\lambda T}}-1}.$$

Derive the corresponding Wien's Law.

c) The surface temperature of the sun is 5778 K. Find the "most likely" energies of solar radiation according to the two versions of Wien's Law. Why are they **NOT** the same?

### Q2 Particle adventure.

Visit the PDG website link to look up particle properties.

a) Construct a table of the masses of the lowest vector mesons ($1--$ state): $\left(u\bar{u} = d\bar{d}, s\bar{s}, c\bar{c}, b\bar{b}\right)$, alongside the current quark masses.
b) Look up the Fermi coupling constant $G_F$ and mass of W-boson $M_W$.
c) Given

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2},$$

what is the value of the weak interaction coupling constant $g_W$ and $\alpha_W = \frac{g_W^2}{4\pi}$ ? Compare this to the QED value

$$\alpha_{EM} = \frac{1}{137},$$

which one is stronger?

## Q3 Kinematics.

a) Consider the scattering of two particles (of mass $m_1$ and $m_2$) in the Center-of-Mass (COM) frame. The energy in this frame is also the invariant mass $\sqrt{s}$ of the system, i.e.

$$E_{\text{COM}} = \sqrt{s} = \sqrt{q^2 + m_1^2} + \sqrt{q^2 + m_2^2}.$$

Show that

$$q(s) = \frac{1}{2}\sqrt{s}\sqrt{1 - \frac{(m_1 + m_2)^2}{s}}\sqrt{1 - \frac{(m_1 - m_2)^2}{s}}.$$

b) Show that the invariant mass ($\sqrt{s}$) of a 3-body system satisfies

$$s = s_{12} + s_{23} + s_{13} - m_1^2 - m_2^2 - m_3^2,$$

where standard relativistic kinematics apply, i.e.

$$\begin{aligned} s_{ij} &= (p_i + p_j)^2 \\ p_i &= (E_i, \vec{p}_i) \\ p_i^2 &= E_i^2 - (\vec{p}_i)^2. \end{aligned}$$

## Q4 N-body phase space.

The N-body Lorentz Invariant phase space (LISP) is defined as

$$\phi_N(s = P^2) = \int \frac{d^3p_1}{(2\pi)^3}\frac{1}{2E_1}\frac{d^3p_2}{(2\pi)^3}\frac{1}{2E_2}\cdots\frac{d^3p_N}{(2\pi)^3}\frac{1}{2E_N}\times \\ (2\pi)^4\,\delta^4(P - \sum_i p_i).$$

Note that $E_j = \sqrt{p_j^2 + m_j^2}$.

a) Derive an analytic expression of the 2-body phase space. (in terms of $s, m_1, m_2$)

b) For all $m_i = 0$, derive an analytic expression of the 3-body phase space.

ans:

$$\phi_3(s) = \frac{s}{256\pi^3}.$$

## Q5 Muon Decay

The decay width of the muon can be estimated by

$$\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}.$$

a) What is the interaction involved? Draw the Feynman diagram.

b) Understand the expression using dimensional analysis. Plug in numbers and calculate the lifetime.

c) Explain the value of the cutoff in the energy distribution of electron (see Fig. 1).

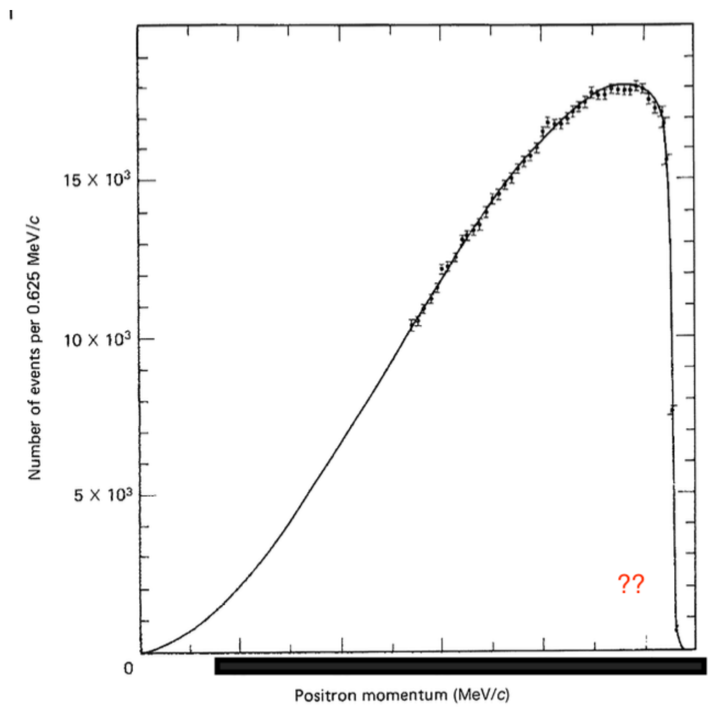


**Fig. 9.1** Experimental spectrum of positrons in $\mu^+ \to e^+ + \nu_e + \nu_\mu$. The solid line is the theoretically predicted spectrum based on Equation (9.33), corrected for electromagnetic effects. (*Source*: Bardon, M. *et al.* (1965) *Physical Review Letters*, **14**, 449. For the latest high-precision data on muon decay go to the TWIST collaboration web site at TRIUMF, Vancouver, BC.)

Figure 1: muon decay