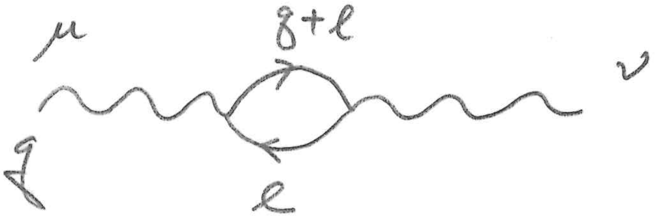
photon polarization

03. 2025

①



Ref.
Peskin & Schröder
Zee.

$$i\Pi^{\mu\nu}(q) = \text{Diagram}$$

$$= (-1)(-ie)^2 \int \frac{d^4 l}{(2\pi)^4} \, tr\left[\gamma^\mu \frac{i}{\not{l}-m} \gamma^\nu \frac{i}{\not{l}+\not{q}-m}\right]$$

$$= -e^2 \int \frac{d^4 l}{(2\pi)^4} \frac{N^{\mu\nu}}{(l^2-m^2)\left[(l+q)^2-m^2\right]}$$

$$N^{\mu\nu} = tr\left[\gamma^\mu (\not{l}+m) \gamma^\nu (\not{l}+\not{q}+m)\right]$$

$$= 4\left\{ l^\mu (l+q)^\nu + l^\nu (l+q)^\mu - g^{\mu\nu}\left[l\cdot(l+q) - m^2\right] \right\}$$

step 1 : Feynman parametrization

$$\frac{1}{\ell^2-m^2}\ \frac{1}{(\ell+q)^2-m^2} = \int_0^1 dx\ \frac{1}{[\ell'^2-\Delta]^2}$$

$$\ell' = \ell + (1-x)q$$

$$\Delta = m^2 - x(1-x)q^2$$

here's how :

$$\frac{1}{AB} = \int_0^1 dx\ \frac{1}{[Ax + B(1-x)]^2}$$

$$A \rightarrow \ell^2 - m^2$$

$$B \rightarrow (\ell+q)^2 - m^2$$

$$x(\ell^2-m^2) + (1-x)[\ell^2 + 2\ell\cdot q + q^2 - m^2]$$

$$= \ell^2 + 2(1-x)\ell\cdot q + (1-x)^2 q^2 - [m^2 - x(1-x)q^2]$$

$$= (\ell + (1-x)q)^2 - (m^2 - x(1-x)q^2)$$

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step 2 : odd man out

by symmetry

$$\int d^4l \; l^\mu l^\nu \longrightarrow \int d^4l \; \frac{1}{4} g^{\mu\nu} l^2$$

$$\int d^4l \; l^\mu \longrightarrow 0$$

Recall
$$l' = l + (1-x) q$$
$$l = l' - (1-x) q$$
$$l + q = l' + x q$$

$$N^{\mu\nu} \to 4 \Big\{ 2 l'^\mu l'^\nu - 2x(1-x) q^\mu q^\nu + \underline{(2x-1)(q^\mu l'^\nu + q^\nu l'^\mu)}$$
$$- g^{\mu\nu} \Big[l'^2 - m^2 - x(1-x) q^2 + \underline{q \cdot l' (2x-1)} \Big]$$

$\underline{\qquad}$ will integrate to 0

also

$$l'^\mu l'^\nu \to \frac{1}{4} g^{\mu\nu} l'^2$$

$$N^{\mu\nu} \longrightarrow 4 \left\{ -\frac{1}{2} g^{\mu\nu} \ell'^2 - 2x(1-x)\, q^\mu q^\nu + g^{\mu\nu}\left(m^2 + x(1-x) q^2 \right) \right\}$$

looks messy

$$\underline{P}^{\mu\nu} = g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}$$

$$N^{\mu\nu} \longrightarrow 4 \left\{ -\frac{1}{2} g^{\mu\nu} \ell'^2 + 2x(1-x)\, q^2\, \underline{P}^{\mu\nu} + g^{\mu\nu}\left[\underline{m^2 - x(1-x) q^2}\right] \right\}$$

Δ appears naturally!

step 3 : 4d Integrals.

$$I_1 = \int \frac{d^4 l}{(2\pi)^4} \frac{1}{(l^2-\Delta)^2}$$

$$I_2 = \int \frac{d^4 l}{(2\pi)^4} \frac{l^2}{(l^2-\Delta)^2}$$

start with I_1

Wick's rotation : $l^0 \to i l_4$.

$$l^2 \to -l_E^2$$

$$I_1 \to \frac{i}{(2\pi)^4} \int d^4 l_E \frac{1}{(l_E^2+\Delta)^2}$$

$$= \frac{i}{16\pi^4} \underbrace{\Omega_{4d}}_{2\pi^2} \underbrace{\int_0^\infty dl_E \, l_E^3 \frac{1}{(l_E^2+\Delta)^2}}_{\text{need to regulate}}$$

Why $\Omega_{4d} = 2\pi^2$?

$$\sqrt{\pi}^4 = \int dx\,dy\,dz\,d\tau \; e^{-\vec{x}^2}$$

$$= \Omega_{4d} \underbrace{\int_0^\infty dR \; R^3 \; e^{-R^2}}_{1/2}$$

$$\Omega_{4d} = 2\pi^2$$

$$\Omega_d = \frac{2\sqrt{\pi}^d}{\Gamma(\frac{d}{2})}$$

$\Gamma(n+1) = n!$

defined even when $n \neq$ integer

Now back to the integral:

$$\int_0^\infty dl_E \frac{l_E^3}{(l_E^2+\Delta)^2}$$

$$= \int_0^1 dL \frac{L^3}{(L^2+\bar{\Delta})^2}$$

$L = l_E/\Lambda$

$\bar{\Delta} = \Delta/\Lambda^2$

$\Lambda \to \infty$

mathematica-lly :

$$\int_0^1 dL \, \frac{L^3}{(L^2+\bar{\Delta})^2} = \frac{1}{2}\left(\frac{-1}{1+\bar{\Delta}} + \ln \frac{1+\bar{\Delta}}{\bar{\Delta}} \right)$$

$$\Lambda \to \infty$$

$$\bar{\Delta} \to 0$$

$$\text{rhs} \longrightarrow -\frac{1}{2}\left(1 + \ln \bar{\Delta}\right).$$

$$I_1 \approx \frac{-i}{16\pi^2}\left(1 + \ln \bar{\Delta}\right).$$

Similarly

$$I_2 = \frac{i}{16\pi^4} \, 2\pi^2 \int_0^\infty dl_E \, \frac{l_E^3 \, (-l_E^2)}{(l_E^2+\Delta)^2}$$

$$= \frac{-i}{16\pi^2} \, 2\Lambda^2 \underline{\int_0^1 dL \, \frac{L^5}{(L^2+\bar{\Delta})^2}}$$

$$1 - \frac{1}{2(1+\bar{\Delta})} - \bar{\Delta} \ln \frac{1+\bar{\Delta}}{\bar{\Delta}}$$

$$\approx \frac{1}{2} + \frac{1}{2}\bar{\Delta} + \bar{\Delta} \ln \bar{\Delta}$$

$$I_2 \approx \frac{-i}{16\pi^2}\left\{ \Lambda^2 + \Delta + 2\Delta \ln \bar{\Delta} \right\}.$$

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Up to now

$$i\Pi^{\mu\nu}(q) = -4e^2\int_0^1 dx \left\{ -\frac{1}{2} g^{\mu\nu} I_2 + 2x(1-x) q^2 P^{\mu\nu} I_1 + \Delta g^{\mu\nu} I_1 \right\}$$

$$\simeq \frac{4ie^2}{16\pi^2}\int_0^1 dx \left\{ -\frac{1}{2} g^{\mu\nu}(\Lambda^2 + 2\Delta \ln\bar{\Delta} + \Delta) + 2x(1-x) q^2 P^{\mu\nu}(1+\ln\bar{\Delta}) + \Delta g^{\mu\nu}(1+\ln\bar{\Delta}) \right\}.$$

$$= i\frac{e^2}{4\pi^2}\int_0^1 dx\, \mathcal{F}^{\mu\nu}.$$

$$\mathcal{F}^{\mu\nu} = \left\{ -\frac{1}{2} g^{\mu\nu}\Lambda^2 + 2x(1-x) q^2 P^{\mu\nu} \ln\bar{\Delta} + g^{\mu\nu}\Delta + 2x(1-x) q^2 P^{\mu\nu} \right\}.$$

by order of div.

Step 5: Pauli - Villars regularization.

ghost fermions to cancel the div

$$J^{\mu\nu} \to J^{\mu\nu}(m) - \sum_a c_a J^{\mu\nu}(M_a)$$

chooses $\sum_a c_a = 1$

$\sum_a c_a M_a^2 =$ m^2

$$J^{\mu\nu}_{(m)} - \sum c_a J^{\mu\nu}(m_a)$$

$$\to 2x(1-x)\, q^2 \, P^{\mu\nu} \Big(\ln \bar{\Delta} - \underbrace{\sum_a c_a \ln \bar{\Delta}_a}_{\ln(M_{PV}^2/\Lambda^2)} \Big)$$

$$\ln \frac{\Delta}{M_{PV}^2}.$$

$$\Pi^{\mu\nu}_{reg} \longrightarrow q^2 P^{\mu\nu} \Pi_{reg}$$

$$\Pi_{reg} = -\frac{e^2}{2\pi^2}\int_0^1 dx \; x(1-x) \ln\frac{m^2 - x(1-x)q^2}{M^2_{PV}}.$$

① the nice thing about $P^{\mu\nu} = g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}$:

$q_\mu P^{\mu\nu} = 0$ by def.

$\Rightarrow q_\mu \Pi^{\mu\nu} = 0$ P.V. reg respecting gauge invariance!

if not (4D reg) $g^{\mu\nu}\Lambda^2$ is disaster!

② how $\Pi^{\mu\nu}$ enters the photon propagator?

$$i D^{(0)}_{\mu\nu} = \frac{-i P_{\mu\nu}}{q^2} = i P_{\mu\nu} D^{(0)}$$

$$i D_{\mu\nu} = i D^{(0)}_{\mu\nu} + i D^{(0)}_{\mu\rho} \, i\Pi^{\rho\sigma} \, i D_{\sigma\nu}$$

$$D = D^{(0)} + D^{(0)}(-q^2\Pi) D$$

$$D^{-1} = D^{(0)-1} + \Pi q^2 = -q^2 + q^2\Pi$$

$$D = \frac{-1}{q^2} \frac{1}{1 - \Pi(q^2)} \quad ''$$

last step (for now) : renormalize

$$e_R^2 = \frac{e^2 \nearrow \text{DIV}}{1-\Pi^{reg}_{(0)} \nearrow \text{DIV}}$$

$\swarrow$

Finite

$$\frac{e^2}{1-\Pi^{reg}(q)} = \frac{e^2}{1-\Pi^{reg}_{(0)}} \frac{1}{1-\frac{1}{1-\Pi^{reg}_{(0)}}\left[\Pi^{reg}_{(q)}-\Pi^{reg}_{(0)}\right]}$$

renormalize the e^2 there!

$$= \frac{e_R^2}{1-\frac{e_R^2}{2\pi^2}\int_0^1 dx \; x(1-x)\ln\frac{M^2-x(1-x)q^2}{M^2}}$$

2 good things :

$\forall$ in terms of e_R^2

$$\hat{\Pi}_R(q) = \frac{1}{1-\Pi^{reg}_{(0)}}\left[\Pi^{reg}_{(q)} - \Pi^{reg}_{(0)}\right]$$

$$= \frac{e_R^2}{2\pi^2}\int_0^1 dx \; x(1-x)\ln\frac{M^2-x(1-x)q^2}{M^2}$$

$\nearrow$

not dependent on M_{PV}^2 or reg. scheme

Running Coupling

$q^2 \to -Q^2$ spacelike.

$$\alpha(Q^2) = \frac{\alpha_0}{1 - \hat{\Pi}_R(Q^2)}$$

$$\hat{\Pi}_R(Q^2) = \frac{2\alpha_0}{\pi} \int_0^1 dx\; x(1-x) \ln\left(1 + x(1-x)\frac{Q^2}{m^2}\right)$$

$$\simeq \begin{cases} \dfrac{\alpha_0}{15\pi} \dfrac{Q^2}{m^2} & \text{small } Q^2 \\[2ex] \dfrac{\alpha_0}{3\pi} \ln \dfrac{Q^2}{\overline{M}^2} & \overline{M}^2 = e^{5/3} m^2 \\ & \text{large } Q^2 \end{cases}$$

to derive these results

$$\int_0^1 dx\; x^2(1-x)^2 = \frac{1}{30}.$$

$$\int_0^1 dx\; x(1-x) \ln x(1-x)\frac{Q^2}{m^2}$$

$$= \frac{1}{18}\left(-5 + 3\ln\frac{Q^2}{m^2}\right)$$

$$= \frac{1}{6} \ln \frac{Q^2}{e^{5/3} m^2}$$

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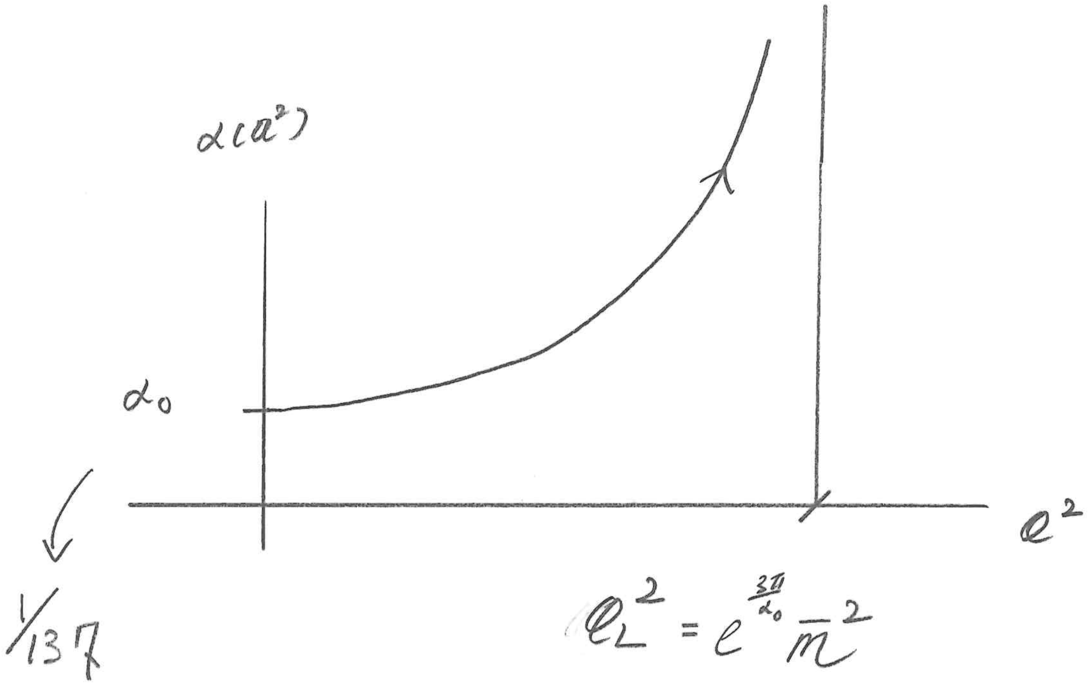


$$q_L^2 = e^{\frac{3\pi}{\alpha_0}} \bar{m}^2$$

Landau pole

$$q_L^2 \sim e^{1300}\ \mathrm{MeV}^2$$

$$|q_L| \sim 10^{280}\ \mathrm{MeV}$$

long way to go ...

RG eqn.

$$\alpha^{-1}(Q^2) = \alpha_0^{-1} - \frac{1}{\alpha_0} \hat{\Pi}_R(Q^2)$$

for large Q

$$\frac{d}{dQ^2} \alpha^{-1} = -\frac{1}{3\pi} \frac{1}{Q^2}$$

$$\beta_\alpha := Q^2 \frac{d}{dQ^2} \alpha = \frac{1}{3\pi} \alpha^2 \quad > 0$$

$\alpha \uparrow$ with $Q \uparrow$.

this is the only info. required from QFT to solve the evolution of coupling

$$\beta_\alpha = \frac{d\alpha}{d \ln Q^2} = \frac{1}{3\pi} \alpha^2$$

$$\Rightarrow \alpha^{-1} = \alpha_{ref}^{-1} - \frac{1}{3\pi} \ln Q^2/Q^2_{ref}$$

$$\alpha = \frac{\alpha_{ref}}{1 - \frac{1}{3\pi} \alpha_{ref} \ln \frac{Q^2}{Q^2_{ref}}}$$

for large Q^2

QED : β-function,

$$\beta_\alpha = \frac{d}{d \ln Q^2} \alpha = \frac{1}{3\pi} \alpha^2$$

or

$$\beta_e = \frac{d}{d \ln Q^2} e = \frac{1}{24\pi^2} e^3$$

phenomenology of vacuum polarization

① Pair creation

$$\hat{\Pi}_R(q) = \frac{e_R^2}{2\pi^2}\int_0^1 dx\, x(1-x)\ln\left(1-x(1-x)\frac{q^2}{m^2}-i\delta\right)$$

$q^2 \to q^2 + i\delta$ by Feynman's prescription.

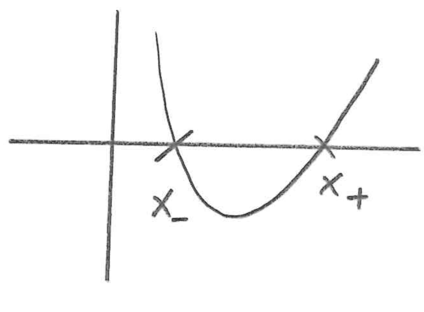
if $q^2 > 4m^2$

$\hat{\Pi}_R \longrightarrow$ imaginary

$$\mathrm{Im}\,\hat{\Pi}_R(q) \xrightarrow[q^2 \to q^2 \pm i\delta]{} \frac{e_R^2}{2\pi^2}\int_0^1 dx\, x(1-x)\,(\mp\pi)\,\theta\left(x(1-x)\frac{q^2}{m^2}-1\right)$$

$$= \mp\frac{e_R^2}{2\pi}\int_{x_-}^{x_+} dx\; x(1-x)$$

$x_\pm$ are to be found via



$$x(1-x)\frac{q^2}{m^2} - 1 > 0$$

$$x^2 - x + \frac{m^2}{q^2} < 0$$

$$x_\pm = \frac{1}{2}\left(1 \pm \sqrt{1-\frac{4m^2}{q^2}}\right)$$

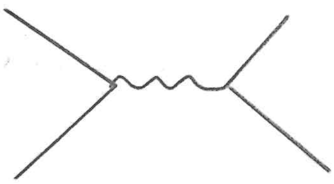
$$\Rightarrow \quad \mathrm{Im}\,\hat{\Pi}_R(q) = \mp \frac{e_R^2}{12\pi}\sqrt{1-\frac{4m^2}{q^2}}\left(1+\frac{2m^2}{q^2}\right)$$

$q^2 \to q^2 \pm i\delta$

$q^2 \geq 4m^2$

Eq. 7.92
P & S.

should take the $-$ sign



$$\sigma_{e^+e^- \to e^+e^-} = \frac{4\pi\alpha^2}{3s}\sqrt{1-\frac{4m^2}{s}}\left(1+\frac{4m^2}{s}\right).$$

$$= -\frac{4\pi\alpha}{s}\,\mathrm{Im}\,\hat{\Pi}_R(s).$$

Eq. 18.86
P & S.

(2) Uehling potential

consider the static limit

$q^2 \to -\vec{q}^{\,2}$ (space like).

$$V(Q^2) = \frac{e^2}{Q^2} \longrightarrow \frac{e^2}{Q^2}\,\frac{1}{1-\Pi(Q^2)}$$

$$\frac{e_R^2}{Q^2}\,\frac{1}{1-\hat{\Pi}_R(Q^2)} \simeq \frac{e_R^2}{Q^2}\left(1+\hat{\Pi}_R(Q^2)\right)$$

$$\hat{\Pi}^R(Q^2) = \frac{e_R^2}{2\pi^2}\int_0^1 dx\; x(1-x)\ln\left(1+x(1-x)\frac{Q^2}{m^2}\right)$$

$$= \frac{e_R^2}{2\pi^2}\begin{cases} \dfrac{1}{30}\dfrac{Q^2}{m^2} & Q^2 \ll m^2 \\[2ex] \dfrac{1}{6}\ln\dfrac{Q^2}{\bar{M}^2} & \bar{M}^2 = e^{5/3}m^2 \\ & Q^2 \gg m^2 \end{cases}$$

if we take the small Q^2 limit

$$V(Q^2) \simeq \frac{e_R^2}{Q^2} + \frac{e_R^4}{60\pi^2}\,\frac{1}{m^2}$$

$$\Rightarrow V(r) = \frac{\alpha_0}{r} + \frac{4\alpha_0^2}{15m^2}\,\delta^3(\vec{x})$$

such $\Delta V \to$ part of Lamb shift :

$$\Delta E \sim -|\psi(0)|^2 \frac{4\alpha_0^2}{15 m^2}$$

attractive pot.

to do it w/o small q^2 approx.

$$V_{\vec{x}} = \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{x}} V(q^2)$$

$$= \frac{1}{2\pi^2 r} \int_0^\infty dq \frac{\sin qr}{q} q^2 V(q^2)$$

$$\to \frac{-i}{4\pi^2 r} \int_{-\infty}^{+\infty} dq \frac{e^{iqr}}{q} q^2 \frac{V(q^2)}{\frac{e_R^2}{q^2}[1+\hat{\Pi}_R(q^2)]}$$

cos qr piece $\to 0$ any way.

$$V_{\vec{x}} = \frac{e_R^2}{4\pi r} + \Delta V(r)$$

$$\Delta V(r) = \frac{-i}{4\pi^2 r} \int_{-\infty}^{+\infty} dq \frac{e^{iqr}}{q} \left(e_R^2 \hat{\Pi}_R(q^2) \right)$$

we derive $\Delta V(r)$ in 2 methods

\#1 Nice trick

Ref.
Jaeok Yi
Schwartz 16.2

$$\Delta V(r) = \frac{-i}{4\pi^2 r}\int_{-\infty}^{+\infty} d\ell \, \frac{e^{i\ell r}}{\ell} \frac{e_R^4}{2\pi^2}\int_0^1 dx$$

$$x(1-x)\ln\left(1 + x(1-x)\frac{\ell^2}{M^2}\right)$$

$$= \frac{-i}{4\pi^2 r}\frac{-e_R^4}{2\pi^2}\int_0^1 dx\left(\frac{1}{2}x^2 - \frac{1}{3}x^3\right)(1-2x)$$

$$\underbrace{\int_{-\infty}^{+\infty} d\ell \, \frac{e^{i\ell r}}{\ell} \frac{\frac{\ell^2}{M^2}}{1 + x(1-x)\frac{\ell^2}{M^2}}}$$

$$i \, e^{\frac{-mr}{\sqrt{x(1-x)}}} \frac{\pi}{x(1-x)}$$

$$= \frac{-e_R^4}{8\pi^3}\frac{1}{r}\int_0^1 dx \left(\frac{1}{2}x^2 - \frac{1}{3}x^3\right)\frac{(1-2x)}{x(1-x)}$$

$$e^{-Mr/\sqrt{x(1-x)}}$$

let's tackle the integral

$$\mathcal{J} = \int_0^1 dx \, \frac{\left(\frac{1}{2}x^2 - \frac{1}{3}x^3\right)(1-2x)}{x(1-x)} \, e^{-\frac{mr}{\sqrt{x(1-x)}}}$$

<u>observe</u>

① more useful to work in variable.

$$u = \frac{1}{2} \frac{1}{\sqrt{x(1-x)}}$$

$$x \in (0, \tfrac{1}{2}) \Rightarrow u \in (\infty, 1)$$

$$x = \frac{1}{2}\left(1 \pm \sqrt{1 - u^{-2}}\right)$$

clearly we pick the lower branch

$$x \longrightarrow \frac{1}{2}\left(1 - \sqrt{1-u^{-2}}\right)$$

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② $$\int_0^1 dx = \int_0^{1/2} dx + \int_{1/2}^1 dx$$

$$x' = 1 - x$$

$x(1-x) \to x'(1-x')$ invariant

$\left(\frac{1}{2}x^2 - \frac{1}{3}x^3\right)(1-2x)$

$\to \left(\frac{1}{6} - \frac{1}{2}x'^2 + \frac{1}{3}x'^3\right)(1-2x')(-1)$

$$\mathcal{I} = \int_0^{1/2} dx \left\{ \frac{-1}{6} + x^2 - \frac{2}{3}x^3 \right\} (1-2x) \frac{1}{x(1-x)} e^{-mr/\sqrt{x(1-x)}}$$

$$du = -\frac{1}{4} \frac{1-2x}{\sqrt{x(1-x)}^3} dx$$

$$\to \int_\infty^1 du \, \frac{-4\sqrt{x(1-x)}}{} \left\{ \frac{-1}{6} + x^2 - \frac{2}{3}x^3 \right\} e^{-2mru}$$

$$\frac{-2}{u} \qquad\qquad \frac{-\sqrt{1-u^{-2}}\,(1+2u^2)}{12u^2}$$

$$= -\int_1^\infty du \, \frac{1}{6}\frac{1}{u^3}\sqrt{1-u^{-2}}\,(1+2u^2)\, e^{-2mru}$$

$$\Delta V(r) = \frac{e_R^4}{48\pi^3}\frac{1}{r} \times$$

$$\int_1^\infty du\, u^{-3}\sqrt{1-u^{-2}}\,(1+2u^2)\, e^{-2mru}$$

if $mr \gg 1$, expand $u \approx 1+t$

$$\Delta V(r) \approx \frac{e_R^4}{48\pi^3}\frac{1}{r} \times$$

$$\underline{\int_0^\infty dt\, \sqrt{2t}\; 3\; e^{-2mr}\, e^{-2mrt}}$$

$$3\sqrt{2}\sqrt{\frac{\pi}{2}}\,\frac{1}{4}\,\frac{1}{\sqrt{mr}^{\,3}}\, e^{-2mr}$$

$$\Delta V \longrightarrow \frac{e_R^4}{16\pi^2}\frac{1}{r}\,\frac{1}{4}\frac{1}{\sqrt{\pi}}\,\frac{e^{-2mr}}{\sqrt{mr}^{\,3}}.$$

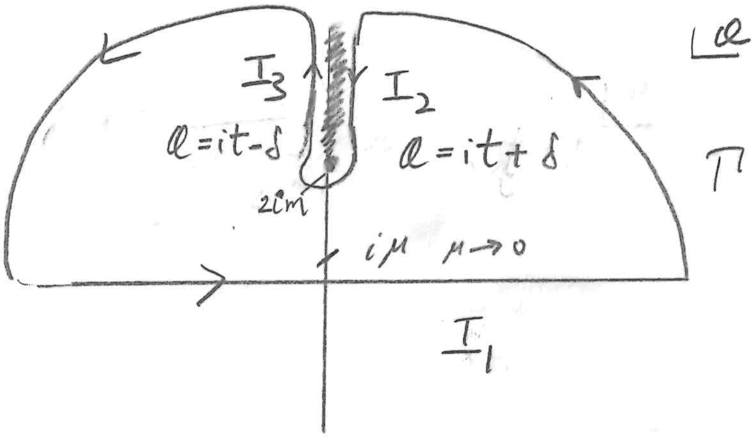
screening

P&S
p. 254 checked!
(we deal w/ the charge)

#2 Physics of branch cut

$$\Delta V(r) = \frac{-i\, e_R^2}{4\pi^2 r}\int_{-\infty}^{+\infty} dq\, \frac{e^{iqr}}{q}\, \hat{\Pi}_R(q^2)$$

$$\frac{e_R^2}{2\pi^2}\int_0^1 dx\; x(1-x)\,\ln\left(1+x(1-x)\frac{q^2}{m^2}\right)$$



the residue at $q = 0 \to 0$

see $\ln 1 \to 0$

$$\mathcal{J} = \oint_\Gamma dq\, \frac{e^{iqr}}{q}\, \hat{\Pi}_R(q)$$

$$\mathcal{J} = 0$$

$$\mathcal{J} = I_1 + (I_2 + I_3)$$

I_1 is what we want!

$$I_1 = -I_2 - I_3$$

$$I_2 = \int_\infty^{2m} dt\, \frac{e^{-tr}}{t}\; \hat{\Pi}_R\; q^2 \to -t^2 + i\delta$$

$$I_3 = \int_{2m}^{\infty} dt \; \frac{e^{-tr}}{t} \; \hat{\Pi}_R (q^2 \to -t^2 - i\delta).$$

$$I_1 = \int_{2m}^{\infty} dt \; \frac{e^{-tr}}{t} \left\{ \hat{\Pi}_R (q^2 \to -t^2 + i\delta) - \hat{\Pi}_R (q^2 \to -t^2 - i\delta) \right\}$$

$$= \int_{2m}^{\infty} dt \; \frac{e^{-tr}}{t} (-2i) \, \underline{\mathrm{Im} \, \hat{\Pi}_R (q^2 \to -t^2 - i\delta)}$$

previously derived

$$\frac{-e_R^2}{12\pi} \sqrt{1 - \frac{4m^2}{t^2}} \times \left(1 + \frac{2m^2}{t^2}\right)$$

$$\Delta V(r) = \frac{e_R^4}{24\pi^3} \frac{1}{r} \int_{2m}^{\infty} dt \; \frac{e^{-tr}}{t} \sqrt{1 - \frac{4m^2}{t^2}} \left(1 + \frac{2m^2}{t^2}\right)$$

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P & S

P. 254 checked.

again if $mr \gg 1$

expand t around. $2m$.

$$t = 2m + t'$$

$$\Delta V(r) \to \frac{e_R^4}{24\pi^3} \frac{1}{r} \left(\frac{3}{8} \frac{\sqrt{\pi}}{\sqrt{mr}^3} e^{-2mr} \right)$$

$$= \frac{e_R^4}{64\pi^2} \frac{1}{r} \frac{1}{\sqrt{\pi}} \frac{e^{-2mr}}{\sqrt{mr}^3}$$

//

agree with
what obtained
before!