QFT

02. 2025

0.6 + 0.4

HWs 4-5

proj : literature review

exam

Tue 1600-1900.

lecture + tutorial

work out problems.

Walecka
Cheng & Li

QFT → focus on phenomenology
→ have to think

Topics : Adv. QM
Spectrum of Hadrons
Functional Integrals
Scattering
Nuclear Physics.
* FTFT.

Why QFT ?

difficult to calculate things

∞

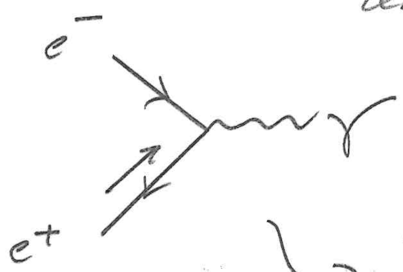
pretty useless.

except that's the only theory that works.

$\exists\ e^+$

particle # can change

QM is not enough...



$\frac{QM}{\Delta E \Delta t \sim 1}$

$\Delta t \downarrow \quad \Delta E \uparrow \quad \sim \quad 2m_e$

QFT $\approx$ QM + rel ✓

$\frac{Rel}{\ }$ $E = mc^2$

condensed matter physics

superconductivity ✓

spin of fermions

$(\vec{L} + 2\vec{S})$ $\longrightarrow$ fine splitting ✓

$=$

$g=2$

Aim of this course :

understand QFT to the point it becomes practical std. model

strong: $\begin{pmatrix} R \\ G \\ B \end{pmatrix}$ $SU(3)$

weak $\begin{pmatrix} n \\ p \end{pmatrix}$ or $\begin{pmatrix} d \\ u \end{pmatrix}$; $\begin{pmatrix} e^- \\ \nu_e \end{pmatrix}$ $SU(2)$.

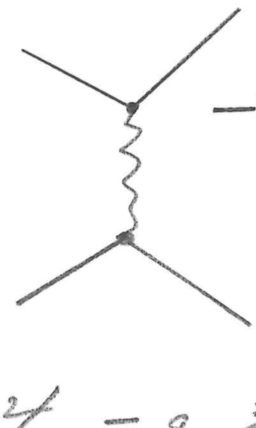
EM e^- $U(1)$

gravity ?

QFT $\rightarrow$ gauge theory $\rightarrow$ gauge symmetry $\rightarrow$ conservation laws

gauge charge

gauge bosons



$\rightarrow -ig\gamma$ — charge = coupling strength

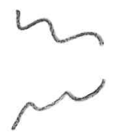
$\rightarrow$ boson exchange.

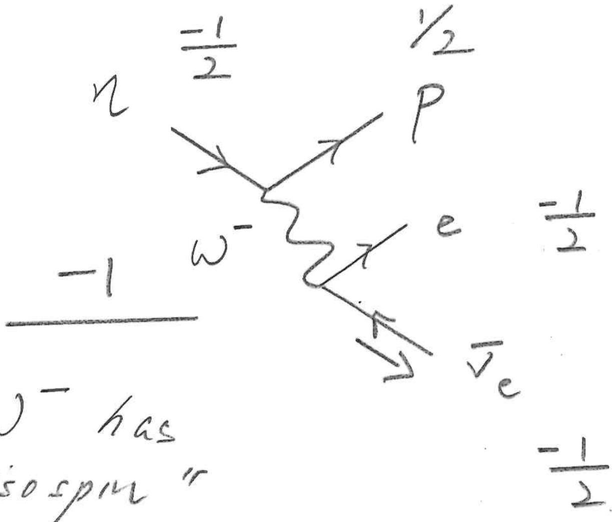
$$\mathcal{L} = \bar{\psi}(i\not{\partial}-m)\psi - g\,\bar{\psi}\not{A}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Group theory : dim. of fundamental repr $\rightarrow$ column vec.

adj. repr : N_c^2-1 $N_c \times N_c$ matrix gauge charge $\leftrightarrow$ interaction.

γ is chargeless

γ
γ  ? don't interact $\longrightarrow$ QFT Box diagram.



$\binom{n}{p}$

$\binom{e}{\nu_e}$

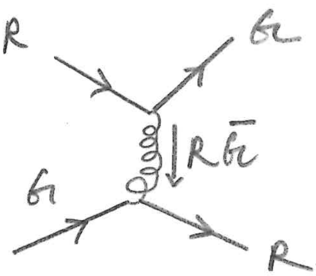
weak isospin

$\bar{\nu}_e$: $\frac{-1}{2}$

W^- has "isospin" charge.

Vs γ

color exch in quark-quark scat.



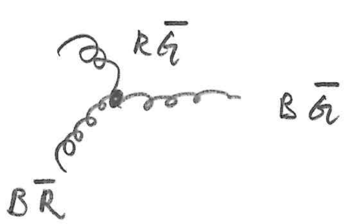
Gluons $\longrightarrow$ $R\bar{G}$

$\Longrightarrow$ abelian Vs non-abelian

U(1) SU(2)

SU(3)

 ? don't exist (at tree level)



World Domination

DoFs	
quarks & gluons	QCD $\rightarrow$ LQCD $\rightarrow$ Spectroscopy ~ Monte Carlo, Sign problem, Pure Gauge; LQCD $\rightarrow$ FTQCD
	QCD $\rightarrow$ Schwinger Dyson Eqn. ~ Continuum, Truncation, functional
	QCD $\rightarrow$ pQCD ~ asymptotic freedom, resummation, NP phy.
π's, N	χ_{pt} — EFTs, LECs — (non)linear σ model, sym.
π, σ, ω	V_{NN} — pot. model, Schrödinger eqn., scattering theory — Walecka model, $\pi, \sigma, \omega, \rho, a_1$ ...
A, Z α, β, γ	Atomic Nucleus — Brückner HF, GFs, Liquid Drop Model, Shell Model
hadrons + stat. mech.	Nuclear Matter, HICs — Walecka, HRG — den. functional, Fermi Gas.
quarks + stat. mech.	Quark Matter — NJL, functional approach.
Universe life time	Astrophysics. — NS, PNS, BHs — EoSs — composition.

DOFs

quarks & gluons

π's

N π σ ω.

A, Z

hadronic phase

quark phase

QCD → LQCD → Spectroscopy; FTQCD ~ Monte Carlo, Sign problem. Pure Gauge

QCD → Schwinger Dyson Eqn. ~ continuum, truncation, functional method

QCD → pQCD ~ asym. freedom, resummation, NP phy.

how to study BS?

χpt ~ QST, LECs; linear σ model, NNLO

V_{NN} ~ potential model, Schrödinger eqn., scattering theory; Walecka model, π, σ, ω, ρ.

Atomic Nucleus ~ Brückner HF, EFs; Liquid Drop Model; Shell model

Nuclear Matter, HICs ~ Walecka, HREL; den. functional, Fermi Gas

Quark Matter ~ NJL, functional approach.

Astrophysics: NS, BNS, BH ~ EOSs, composition

QFT &
Many body techniques.

Eff.
models

χ quark
model
&
gluon
model

potential
models &
scattering theory

kinematics & dynamics.

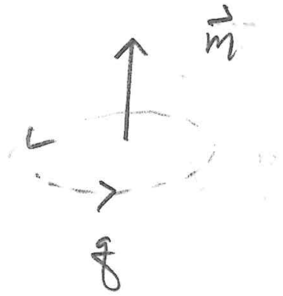
3 ways to QFT
path integral = quantization

basic QFT.
scalar, fermion, gauge fields.
Sym. & consequences.

χ sym.
low E theorems.
SSB.
Goldstone
bosons
etc.

Functional techniques.
Generating functionals.
Eff. actions
density functional.

Angular Momentum & Spins



current $I \rightarrow q/\tau$

$\swarrow$ period

$$\vec{L} = \vec{r} \times \vec{p}$$

$$= mvr\,\hat{z}$$

$$v = \frac{2\pi}{\tau}$$

$$\vec{m} = I\vec{A}$$

$$= \frac{q}{\tau}\pi r^2\,\hat{z}$$

$$= \frac{1}{2m}q\,\vec{L} \qquad \underline{\vec{m} \propto \vec{L}}$$

$\frac{e}{2m} \rightarrow$ Bohr magneton.

$$\Rightarrow \vec{m} \propto\ ?\ \vec{S}$$

$$\vec{m} = \frac{q}{2m}\left(\vec{L} + 2\vec{S}\right)$$

we can show by QFT that

$$H_{int} = -\frac{q}{2m}(\vec{L} + \underline{\underline{2}}\vec{S}) \cdot \vec{B}$$

g-fac.

phenomenological potential in QM:

$$H = \frac{p^2}{2M_R} + V$$

$$V = V_1(r) + V_2(r)\, \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

$$+ V_3\, \vec{L} \cdot \vec{S} +$$

tensor structure $\leftarrow$ $$V_4 \left(3 \frac{\vec{\sigma} \cdot \vec{r}\ \ \vec{\sigma} \cdot \vec{r}}{r^2} - \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right)$$

$\vec{J} = \vec{L} + \vec{S}$ is conserved.

$\vec{J}^2 \to j(j+1)$ j is a good Q#

$$\vec{S} = \frac{1}{2}\vec{\sigma}$$

$$\vec{\sigma}_1 \cdot \vec{\sigma}_2 \rightarrow 4\, \vec{S}_1 \cdot \vec{S}_2$$
$$= 4\, \frac{1}{2} \left[(\vec{S}_1 + \vec{S}_2)^2 - \vec{S}_1^{\,2} - \vec{S}_2^{\,2} \right]$$
$$= 2\vec{S}^2 - 3\,\mathbb{I}$$

spin-orbit coupling

$$\vec{L} \cdot \vec{S} \rightarrow \frac{1}{2} \left(\vec{J}^2 - \vec{L}^2 - \vec{S}^2 \right)$$

$$S_{12} = \left(3\, \vec{\sigma}_1 \cdot \hat{r}\, \vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right)$$
$$\rightarrow 2 \left(3 \frac{(\vec{S} \cdot \vec{r})^2}{r^2} - \vec{S}^2 \right)$$

$\sim$ rank 2 tensor

Messiah
P. 554
P. 580

$\Rightarrow$ how to get these in QCD ?

$$V_\pi \rightarrow \frac{f_\pi^2}{4\pi m_\pi^2}\, \vec{\tau}_1 \cdot \vec{\tau}_2\ \vec{\sigma}_1 \cdot \nabla_1\ \vec{\sigma}_2 \cdot \nabla_2 \times$$

OPEP

$$\frac{e^{-m_\pi r}}{r}$$

exercise :

$$\vec{S}_1^{\,2} = \frac{1}{4}\,\vec{\sigma}_1^{\,2} = \frac{1}{4}\,3\,\mathbb{I} = \frac{3}{4}\,\mathbb{I} \leftrightarrow \frac{1}{2}\left(\frac{1}{2}+1\right)\mathbb{I}$$

$$\vec{\sigma}_1\cdot\vec{x}\;\;\vec{\sigma}_1\cdot\vec{x}$$

$$= \sigma_1^i x^i \;\; \sigma_1^j x^j$$

symmetric

$$= \frac{1}{2}\Big(\underbrace{\sigma_1^i\sigma_1^j + \sigma_1^j\sigma_1^i}_{2\delta^{ij}}\Big)\;x^i x^j$$

$$\longrightarrow \vec{x}^{\,2}$$

$$S_{12} = 3\,\frac{\vec{\sigma}_1\cdot\vec{x}\;\;\vec{\sigma}_2\cdot\vec{x}}{|\vec{x}|^2} - \vec{\sigma}_1\cdot\vec{\sigma}_2$$

$$= 4\left\{ 3\,\frac{\vec{S}_1\cdot\vec{x}\;\;\vec{S}_2\cdot\vec{x}}{|\vec{x}|^2} - \vec{S}_1\cdot\vec{S}_2 \right\}$$

$$= 4\left\{ 3\,\frac{1}{2}\,\frac{(\vec{S}\cdot\vec{x})^2 - (\vec{S}_1\cdot\vec{x})^2 - (\vec{S}_2\cdot\vec{x})^2}{|\vec{x}|^2} - \vec{S}_1\cdot\vec{S}_2 \right\}$$

$$= 4\left\{ 3\,\frac{1}{2}\,\frac{(\vec{S}\cdot\vec{x})^2}{\vec{x}^{\,2}} - \frac{3}{4}\,\mathbb{I} - \frac{1}{2}\left(\vec{S}^{\,2} - \vec{S}_1^{\,2} - \vec{S}_2^{\,2}\right) \right\}$$

$$\longrightarrow 2\left\{ 3\,(\vec{S}\cdot\hat{r})^2 - \vec{S}^{\,2} \right\}$$

$$\longrightarrow \sqrt{\frac{24\pi}{5}}\;\left[S^{(2)}\cdot Y^{(2)}\right]$$

03.2025

Tensor construction

$A_i B_j$ 9 entries

$$A_i B_j = \frac{1}{2}(A_i B_j + A_j B_i) + \frac{1}{2}[A_i B_j - A_j B_i]$$

$$= \left(\frac{1}{3}\vec{A}\cdot\vec{B}\,\delta_{ij} + a_{ij}\right) + \quad \text{Sym.}$$

$$\frac{1}{2}\varepsilon_{ijk}(\vec{A}\times\vec{B})_k \quad \text{Anti-Sym.}$$

where

$$a_{ij} = \frac{1}{2}\left[A_i B_j + A_j B_i - \frac{2}{3}(\vec{A}\cdot\vec{B})\,\delta_{ij}\right]$$

We identify 3 structures of the tensor

$$R_q^{(0)} \sim \delta_{ij}\, r^2 \qquad 9 = 1 + 3 + 5$$

$$R_q^{(1)} \sim \varepsilon_{ijk}\, r^k$$

$$R_q^{(2)} \sim \frac{1}{2}(r_i r_j + r_j r_i) - \frac{1}{3}\vec{r}^{\,2}\delta_{ij}$$

when we combine 2 tensors :

$$R_q^{(0)} \sigma_q^{(0)} \sim r^2 \vec{\sigma}\cdot\vec{\sigma}$$

$$R_q^{(1)} \sigma_q^{(1)} \sim \vec{r}\cdot\vec{\sigma}$$

$$R_q^{(2)} \sigma_q^{(2)} \rightarrow \sum_q R_q^{(2)} \sigma_q^{(2)} (-1)^q .$$

$$\sim \left(\frac{r_i r_j + r_j r_i}{2} - \frac{1}{3} r^2 \delta_{ij}\right)\left(\frac{\sigma_i\sigma_j + \sigma_j\sigma_i}{2} - \frac{1}{3}\vec{\sigma}^2 \delta_{ij}\right)$$

$$= \vec{r}\cdot\vec{\sigma}\ \vec{r}\cdot\vec{\sigma} - r^2\vec{\sigma}^2\left(\frac{2}{3} - \frac{1}{9}3\right)$$

$$= \vec{r}\cdot\vec{\sigma}\ \vec{r}\cdot\vec{\sigma} - \frac{1}{3} r^2 \sigma^2$$

//

Zeeman effects:

P. 312
Modern Physics.
Serway Moses
Moyer.

$$\mathcal{U} = -\vec{m}\cdot\vec{B}$$

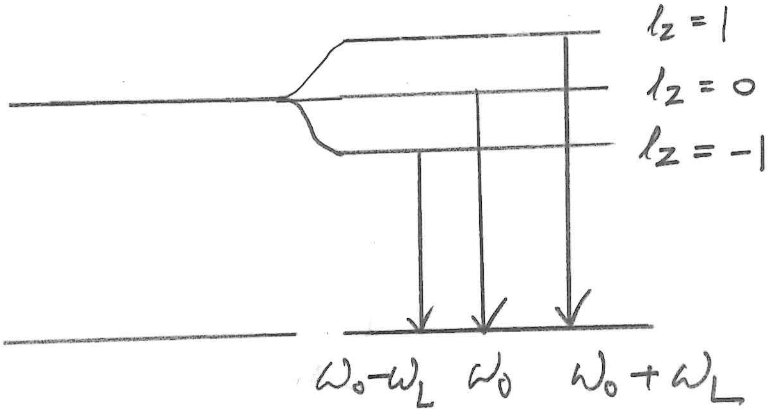
$$\vec{m} = \frac{-e}{2m_e}\vec{L}$$

$$\mathcal{U} \to \frac{e}{2m_e} L_z B_z = L_z\,\omega_L$$

Larmor frequency

$n = 2,\ l = 1$

$n = 1,\ l = 0$



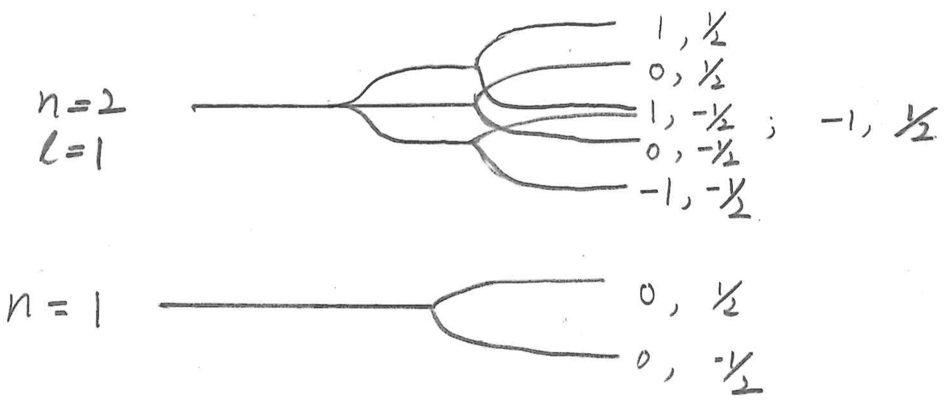
$$\vec{m} \to \frac{-e}{2m_e}(\vec{L} + 2\vec{S})$$

g-factor

$$\mathcal{U} \to \omega_L(l_z + 2S_z)$$

l_z: $-1, 0, 1$

S_z: $\frac{-1}{2}, \frac{1}{2}$



$$E_\gamma = \left(\omega_0 \pm 3\omega_L,\ \omega_0 \pm 2\omega_L,\ \omega_0 \pm \omega_L,\ \omega_0\right)$$

$1, \frac{1}{2} \to 0, -\frac{1}{2}$

$-1, -\frac{1}{2} \to 0, +\frac{1}{2}$

work out the rest!

$\Rightarrow$ extra constraint from $\vec{J} = \vec{L} + \vec{S}$ conservation:

$\Delta m = -1, 0, 1.$

$m = l_z + s_z$:

$l = 1$ states

$-\frac{3}{2}$ $-\frac{1}{2}$ $+\frac{1}{2}$ $+\frac{3}{2}$

$-\frac{1}{2}$ $\frac{1}{2}$

$l = 0$ states

$-\frac{1}{2}$ $\frac{1}{2}$

the constraint kills $\longrightarrow \omega_0 \pm 2\omega_L$.

only $10/12$ is allowed.

$\omega_0,\ \omega_0 \pm \omega_L,\ \omega_0 \pm 2\omega_L$

QM refresher

$E \to i\partial_t \qquad p \to -i\nabla$

$$i\partial_t \psi = \frac{p^2}{2m}\psi$$

from $\psi \to \rho = \psi^*\psi$

$$\partial_t \rho = \dot{\psi}^*\psi + \psi^*\dot{\psi}$$

$$= \left(-i\frac{\nabla^2}{2m}\psi^*\right)\psi + \psi^* + i\frac{\nabla^2}{2m}\psi$$

$$= -\nabla \cdot \left\{ \frac{-i}{2m} \psi^* \overleftrightarrow{\nabla} \psi \right\}.$$

$$\partial_t \rho + \nabla \cdot \vec{J} = 0$$

$$(\rho, \vec{J}) = \left(\psi^*\psi, \frac{-i}{2m}\psi^*\overleftrightarrow{\nabla}\psi\right).$$

$$E^2 = \vec{p}^2 + m^2$$

$$-\partial_t^2 = -\nabla^2 + m^2$$

But ...

$$(\partial^2 + m^2)\psi = 0.$$

2nd order in ∂_t^2 is problematic

$\psi_{t_i} \rightarrow \psi_{t_i + \Delta t}$ no longer dictates

$$j^\mu = \begin{cases} \frac{i}{2m} \psi^* \overleftrightarrow{\partial_t} \psi \\ \frac{-i}{2m} \psi^* \overleftrightarrow{\nabla} \psi \end{cases}$$

$$\rightarrow \frac{i}{2m} \psi^* \overleftrightarrow{\partial^\mu} \psi$$

$$\partial_\mu j^\mu = \frac{i}{2m} \partial_\mu \left(\psi^* \partial^\mu \psi - (\partial^\mu \psi^*) \psi \right)$$

$$= 0 \quad //$$

$\rho = j^0 \rightarrow$ involve $\partial_t \psi$

$\psi \sim e^{\pm i E t} \rightarrow e^{+iEt} \Rightarrow \rho < 0$

trouble!