Why QFT ?

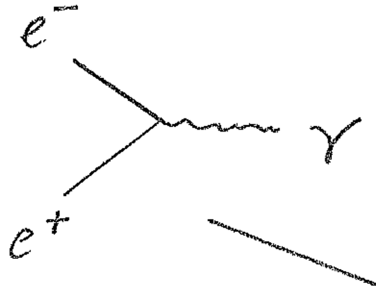
pretty useless —— difficult to calculate things
∞ most of the time

except that's the only theory that works

e^+ exists ✓

particle # can change ✓

QM is not enough



→ QM :

$\Delta E \, \Delta t \sim 1$

$\Delta t \downarrow \quad \Delta E \uparrow \sim 2m_e$

↙ Relativity :

$E = mc^2$

QFT = QM + Rel.

↙

other flavor : condensed matter physics

→ superconductivity ✓

spin of fermions

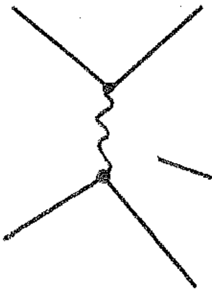
$(\vec{L} + 2\vec{S}) \cdot \vec{B}$ → fine splitting

↳ $g = 2$ ✓

our goal :

understand QFT to a point that it becomes practical !

Gauge theory



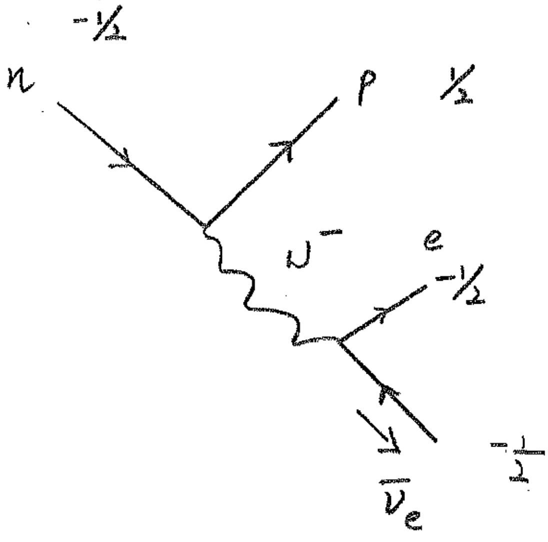
$\rightarrow \quad -ig\gamma$ → gauge charge; γ → Dirac matrix

$\rightarrow \quad iD(p) = \frac{-i\,g_{\mu\nu}}{p^2}$ Gauge Bosons exchange

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi - g\,\bar{\psi}\not{A}\psi$$
$$- \frac{1}{4} F_{\mu\nu}F^{\mu\nu}$$

Group theory ⟶ symmetry : SU_2 SU_3 $U(1)$

fundamental repr ; adj repr



Gp. structure (isospin)

W^- : isospin charge -1
adj

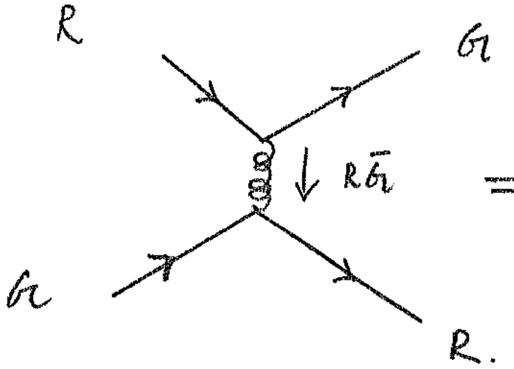
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}$ $\quad \begin{pmatrix} p \\ n \end{pmatrix}$ funda.

$\rightarrow +\frac{1}{2}$

$\quad -\frac{1}{2}$

$\sigma_z = \begin{bmatrix} 1 & \\ & -1 \end{bmatrix}$

Gluons ? $R\bar{G}$

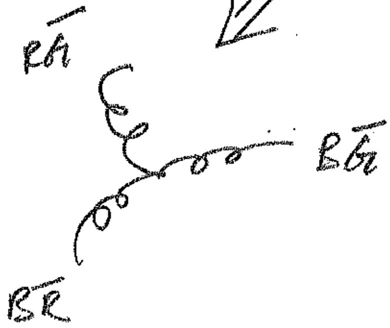


⇒ Gluons have colors !

→ they are <u>charged</u>

↙ Non-abelian gauge theory

⇙



QM
refresher

$E \to i\partial_t \qquad p \to -i\nabla$

$$i\partial_t \psi = \frac{p^2}{2m}\psi$$

$\swarrow$

from $\psi \to \rho = \psi^* \psi$

$$\partial_t \rho = \dot{\psi}^* \psi + \psi^* \dot{\psi}$$

$$= \left(-i \frac{\nabla^2}{2m} \psi^*\right) \psi + \psi^* + i \frac{\nabla^2}{2m} \psi$$

$$= -\nabla \cdot \left\{ \frac{-i}{2m} \psi^* \overleftrightarrow{\nabla} \psi \right\}.$$

$$\partial_t \rho + \nabla \cdot \vec{j} = 0$$

$$(\rho, \vec{j}) = \left(\psi^* \psi, \frac{-i}{2m} \psi^* \overleftrightarrow{\nabla} \psi\right).$$

$$E^2 = \vec{p}^{\,2} + m^2$$

$$-\partial_t^2 = -\nabla^2 + m^2$$

$$(\partial^2 + m^2)\psi = 0 . \quad \nearrow$$

But ...

2nd order in ∂_t^2 is problematic

$\psi_{t_i} \longrightarrow \psi_{t_i + \Delta t}$

no longer dictates the phase of $e^{-iE\Delta t}$

↙

$$j^\mu = \begin{cases} \frac{i}{2m}\, \psi^* \overleftrightarrow{\partial_t}\, \psi \\ \frac{-i}{2m}\, \psi^* \overleftrightarrow{\nabla}\, \psi \end{cases}$$

$$\longrightarrow \frac{i}{2m}\, \psi^* \overleftrightarrow{\partial^\mu}\, \psi$$

$$\partial_\mu j^\mu = \frac{i}{2m}\, \partial_\mu \left(\psi^* \partial^\mu \psi - (\partial^\mu \psi^*) \psi \right)$$

$$= 0$$

//

$\rho = j^0 \longrightarrow$ involve $\partial_t \psi$

$\psi \sim e^{\pm i E t} \longrightarrow$ e^{iEt} component is NOT avoidable

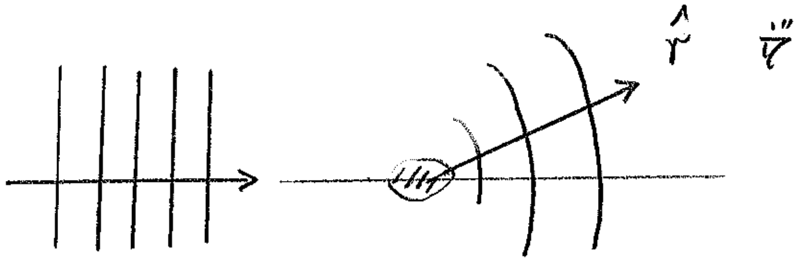
$$\rho \longrightarrow \frac{i}{2m}\, e^{-iEt} \overleftrightarrow{\partial_t}\, e^{iEt}$$

$$= \frac{-E}{m} < 0$$

02.2026

Scattering Theory

①



$$E_q = \frac{q^2}{2M_r}$$

e^{iqz} $\quad\quad$ $\frac{e^{iqr}}{r} f_q$ $\quad$ spherical waves

f_q : scattering amplitude

$$\psi_{\vec{x}} \xrightarrow{r\to\infty} e^{iqz} + \frac{e^{iqr}}{r} f_q$$

$$H|\psi\rangle = (H_0 + V)|\psi\rangle = E|\psi\rangle$$

$$(E - H_0)|\psi\rangle = V|\psi\rangle$$

$$\Rightarrow \quad |\psi\rangle = |\varphi\rangle + \frac{1}{E - H_0} V|\psi\rangle$$

↙ satisfies

$$(E - H_0)|\varphi\rangle = 0$$

$$\langle\vec{x}|\psi\rangle = \langle\vec{x}|\varphi\rangle + \langle\vec{x}|\frac{1}{E-H_0} V|\psi\rangle$$

↙ $\psi_{\vec{x}}$ $\quad$ ↙ e^{iqz} $\quad$ ↙

$$\int d^3x' \, \langle\vec{x}|\frac{1}{E-H_0}|\vec{x}'\rangle V_{\vec{x}'} \psi_{\vec{x}'}$$

$$G^0_{\vec x \vec x'} = \langle \vec x | \frac{1}{E - H_0 \underline{+ i\delta}} | \vec x' \rangle$$

$$H_0 = \frac{-\nabla^2}{2m_r}$$

↳ Retarded Green's function

$$\Delta\psi_{scat} = \int d^3\vec x' \, G^{(0)}_{\vec x \vec x'} V_{\vec x'} \psi_{\vec x'}$$

$$\leftrightarrow \frac{e^{ikr}}{r} f$$

$$G^0_{\vec x \vec x'} = \int \frac{d^3p'}{(2\pi)^3} \frac{1}{E - \frac{p'^2}{2m_r} + i\delta} e^{i\vec p' \cdot (\vec x - \vec x')}$$

$$= 2m_r \frac{-1}{4\pi |\vec x - \vec x'|} e^{i\underline{q} |\vec x - \vec x'|}$$

↳ $E = \frac{q^2}{2m_r}$

see Ex.
for proof

$$\psi_{\vec x} = e^{iqz} + \int d^3\vec x' \frac{-2m_r}{4\pi |\vec x - \vec x'|} e^{iq|\vec x - \vec x'|} V_{\vec x'} \psi_{\vec x'}$$

$$\xrightarrow{r \to \infty} e^{iqz} + \frac{e^{iqr}}{r} \underline{\int d^3\vec x' \frac{-2m_r}{4\pi} e^{-iq\hat x \cdot \vec x'} V_{\vec x'} \psi_{\vec x'}}$$

$$\updownarrow$$

$$f_q$$

Born approx.

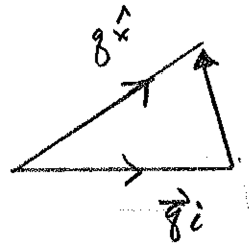
$\psi_{\vec{x}'}$ in the integral by $\varphi_{\vec{x}'} = e^{i\vec{q}_i\cdot\vec{x}'}$

$\swarrow$ along $\hat{z}$

full formula :

$$f_q = \frac{-2M_r}{4\pi}\int d^3x' \, e^{-iq\hat{x}\cdot\vec{x}'} \, V_{\vec{x}'} \, \psi_{\vec{x}'}$$

$$\xrightarrow{\text{Born}} \frac{-2M_r}{4\pi}\int d^3x' \, e^{-iq\hat{x}\cdot\vec{x}'} \, V_{\vec{x}'} \, e^{i\vec{q}_i\cdot\vec{x}'}$$



$\Delta\vec{q} = q\hat{x} - \vec{q}_i$

$$= \frac{-2M_r}{4\pi}\underline{\int d^3x' \, e^{-i\Delta\vec{q}\cdot\vec{x}'} \, V_{\vec{x}'}}$$

FT of $V_{\vec{x}}$

to $\Delta\vec{q}$ space.

$$= \frac{-2M_r}{4\pi}\tilde{V}_{\Delta\vec{q}}$$

//

Generally

$$f_q = \frac{-2M_r}{4\pi}\int d^3x' \, e^{-iq\hat{x}\cdot\vec{x}'} \, V_{\vec{x}'} \, \psi_{\vec{x}'}$$

$$= \frac{-2M_r}{4\pi}\langle \vec{q}_f | V | \psi \rangle$$

//

T-matrix POV

$$|\psi\rangle = |\varphi\rangle + \frac{1}{E-H_0} V|\psi\rangle$$

$$V|\psi\rangle = V|\varphi\rangle + V\frac{1}{E-H_0}V|\psi\rangle.$$

$$= V|\varphi\rangle + V\frac{1}{E-H_0}V|\varphi\rangle + V\frac{1}{E-H_0}V\frac{1}{E-H_0}V|\varphi\rangle + \cdots$$

$$= \underline{\left(V + V\hat{G}_0 V + V\hat{G}_0 V\hat{G}_0 V + \cdots\right)}|\varphi\rangle$$

$$T := \frac{V}{1-\hat{G}_0 V}$$

$$\swarrow$$

$$T = V + V G_0 T$$

Lippmann-Schwinger Equ.

$$\underline{V|\psi\rangle} = \underline{T}|\varphi\rangle$$

full ψ to be solved $\nearrow$

$\hookrightarrow$ T-matrix to be solved.

$$|\psi\rangle = |\vec{q}_i\rangle + \frac{1}{E-H_0} T |\vec{q}_i\rangle.$$

$$\psi_{\vec{x}} = e^{i\vec{q}_i\cdot\vec{x}} + \int d^3x'\, G^0_{\vec{x}\vec{x}'} \langle \vec{x}'|T|\vec{q}_i\rangle.$$

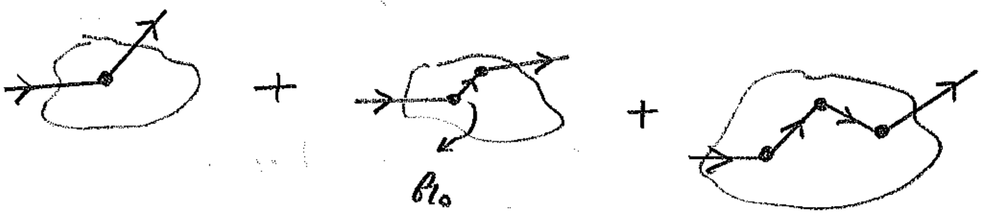
$$\xrightarrow{r\to\infty} e^{i\vec{q}_i\cdot\vec{x}} + \frac{e^{iqr}}{r}\langle \vec{q}_f|T|\vec{q}_i\rangle \left(\frac{-2m_r}{4\pi}\right)$$

Born approx : $T \rightarrow V$

Fourier transform

$$\Rightarrow \quad \langle \vec{q}_f | V | \vec{q}_i \rangle = \tilde{V}(\vec{q}_f - \vec{q}_i)$$

$$T = V + V G_0 V + V G_0 V G_0 V + \dots$$



G POV

$$\hat{G}_0 = \frac{1}{E - \hat{H}_0} \quad \longrightarrow \quad \hat{G} = \frac{1}{E - \hat{H}}$$

Resolvent

$$G^{-1} = G_0^{-1} - V$$

$$G = (G_0^{-1} - V)^{-1}$$

$$= G_0 + G_0 V G_0 + G_0 V G_0 V G_0 + \dots$$

$$\Rightarrow \quad G = G_0 + G_0 \underline{\underline{T}} G_0$$

full GFs. ↑ bare. ↳ full T-matrix

$$A_E = -2 \, \mathrm{Im} \, G_E$$

although V is real.

$T \rightarrow$ complex to satisfy the optical theorem

$$f = \frac{1}{q} \sin\delta \, e^{i\delta}$$

$$\mathrm{Im} f = \frac{\sin^2\delta}{q} = q\,|f|^2$$

how ?

$$\mathrm{Im}\, G_0 \approx 0 \quad \text{but} \quad \mathrm{Im}\, G \neq 0$$

$$G = G_0 + G_0 V G_0 + G_0 V G_0 V G_0 + \cdots$$

---

resum to get a width

$$T_{NR} \longleftrightarrow \frac{-4\pi}{2m_r} f$$

$$S = e^{2i\delta} = 1 + 2i\, q f$$

$$= 1 + i\, \rho_{NR} \frac{4\pi}{2m_r} f$$

$$= 1 - i\, \rho_{NR} T_{NR}$$

$\swarrow$

$$\rho_{NR} = \frac{m_r q}{\pi}$$

$$= \int \frac{d^3 q'}{(2\pi)^3}\, 2\pi\, \delta_{E - \frac{q'^2}{2m_r}}$$

$$= \mathrm{tr}\; 2\pi\, \delta_{E - \hat{H}_0}$$

$$\hat{S} = \hat{I} - i\, 2\pi\, \delta_{E - H_0} \hat{T}$$

$$\hat{S}_{QFT} = \hat{I} \underline{+} i\, (2\pi)^4\, \delta^4_{\underline{p}_f - \underline{p}_i}\, M_{QFT}$$

tragic!
scat theory
vs
P&S QFT

$iM_{QFT} \longleftrightarrow$ Feynman rule.

$M \longleftrightarrow -T$

$\longrightarrow$ Born amplitude

Optical Theorem

02. 2026

$$f(\theta) = \frac{-2m_r}{4\pi} \langle \vec{q}_f | T | \vec{q}_i \rangle$$

Sakurai
P. 391

$$\theta \to 0 \leftrightarrow \vec{q}_f \to \vec{q}_i$$

$$f_{(\theta=0)} = \frac{-2m_r}{4\pi} \langle \vec{q} | T | \vec{q} \rangle$$

forward amplitude

The optical theorem relates the imaginary part of f to $|f|^2$:

$$\operatorname{Im} f_{\theta=0} = q \frac{1}{4\pi} \int d\Omega \, |f_{(\theta)}|^2$$

to prove it :

$$T_{\vec{q}\vec{q}} = \langle \vec{q} | T | \vec{q} \rangle = \langle \vec{q} | V | \psi \rangle$$

$$= \left(\langle \psi | - \langle \psi | V^+ \hat{G}_0^+ \right) \left(V | \psi \rangle \right)$$

$$\hat{G}_0 = \frac{1}{E - \hat{H}_0 + i\delta}$$

$$\longrightarrow \langle \psi | V | \psi \rangle - \langle \psi | V^+ \hat{G}_0^+ V | \psi \rangle$$

$$= \langle \psi | V | \psi \rangle - \langle \vec{q} | T^+ \hat{G}_0^+ T | \vec{q} \rangle$$

V is real $\qquad\qquad \mathrm{Im}\,\hat{G}_0^+ = \pi\,\delta_{E-\hat{H}_0}$

$$\mathrm{Im}\,T_{\vec{q}\vec{q}} = -\,\mathrm{Im}\,\langle\vec{q}|\,T^\dagger\,\hat{G}_0^{+}\,T\,|\vec{q}\rangle$$

$$\to -\frac{1}{2}\int\frac{d^3q'}{(2\pi)^3}\,2\pi\,\delta_{E-\frac{q'^2}{2m_r}}\;T^\dagger_{\vec{q}\vec{q}'}\;T_{\vec{q}'\vec{q}}$$

$$= -\frac{1}{2}\int d\varepsilon\;|T|^2$$

or in terms of f

$$\mathrm{Im}\,f_{\theta=0} = \frac{-2m_r}{4\pi}\left(-\frac{1}{2}\right)\int\frac{d^3q'}{(2\pi)^3}\,2\pi\,\delta_{E-\frac{q'^2}{2m_r}}\;T^\dagger_{qq'}T_{q'q}$$

$$= q\;\frac{1}{4\pi}\int d\Omega\;|f(\theta)|^2 \qquad //$$

This suggests a parametrization.

$$f(\theta) = \sum_\ell (2\ell+1)\,P_\ell(\cos\theta)\,f_\ell$$

$$f_\ell = \frac{e^{i\delta_\ell}}{q}\sin\delta_\ell$$

we can verify

$$\int d\Omega\,|f|^2 = \sum_{\ell\ell'}(2\ell+1)(2\ell'+1)\;2\pi\int_{-1}^{1}dz\;P_\ell P_{\ell'}\;f^*_{\ell'}f_\ell$$

$$= \sum_\ell (2\ell+1)\;4\pi\;|f_\ell|^2 = \frac{4\pi}{q}\;\mathrm{Im}\,f_{\theta=0}\;//$$

$$\psi_q \xrightarrow{r\to\infty} e^{iqz} + \frac{e^{iqr}}{r} f_q.$$

$$\vec{J} = \frac{-i}{2m_r}\, \psi^* \overleftrightarrow{\nabla} \psi$$

$$= \frac{1}{m_r}\, \mathrm{Im}\left(\psi^* \nabla \psi\right)$$

$$\simeq m_r^{-1}\, \mathrm{Im}\left\{ \left[e^{-iqz} + \frac{1}{r} e^{-iqr} f^*\right] \left[iq\hat{z}\, e^{iqz} + i\,(q + ir^{-1})\, \hat{r} \frac{e^{iqr}}{r} f \right] \right\}$$

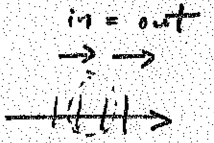
$$= \frac{q}{m_r}\left[\underset{\substack{\uparrow \\ \hat{J}_{in}}}{\hat{z}} + \underset{\substack{\uparrow \\ \hat{J}_{scat}}}{\frac{1}{r^2}|f|^2\, \hat{r}} + \underset{\substack{\uparrow \\ \hat{J}_{interference}}}{(\hat{z}+\hat{r})\, \mathrm{Re}\, \frac{f}{r} e^{iq(r-z)}} \right]$$

$$\nabla\cdot\vec{J} + \partial_t \rho = 0$$

$$\underline{\int d^3x\, \nabla\cdot\vec{J}} = -\partial_t \int d^3x\, \rho = 0.$$

$$\oint \vec{J}\cdot d\vec{A} = 0.$$

$$\oint \vec{J}_{in} \cdot d\vec{A} = 0$$



$$\oint \vec{J}_{scat} \cdot d\vec{A} = \int d\Omega \; r^2 \, \frac{1}{r^2} |f|^2$$

$$\frac{q}{m_r}$$

$$\downarrow$$

$$j_{in}.$$

$$= j_{in} \underbrace{\int d\Omega \; |f|^2}_{\sigma_{scat}.}$$

$$\oint \vec{J}_{interference} \cdot d\vec{A} = \frac{q}{m_r} \, 2\pi \int d\cos\theta \; r^2 (1 + \cos\theta) \times$$

$$Re \, \frac{f}{r} \, e^{iqr(1-\cos\theta)}$$

$$= \frac{q}{m_r} \, 2\pi r \underline{\int_{-1}^{1} dz' \; (1+z') \, Re \, f e^{iqr(1-z')}}$$

$kr \to \infty$

only z': $1-\varepsilon$ to 1 contribute

$$= (j_{in}) \left[\frac{-4\pi}{q} \, Im f_{\theta=0} \right]$$

$$// \quad \approx 2 \int_{1-\varepsilon}^{1} dz' \; Re \, f_{\theta=0} \, e^{iqr(1-z')}$$

$$= 2 \, Re \left[\frac{f_{\theta=0}}{-iqr} \left(1 - \underset{\substack{\uparrow \\ kr \to \infty \\ \to 0}}{e^{iqr\varepsilon}}\right) \right]$$

$$= - \frac{2 \, Im f_{\theta=0}}{qr}$$

the last 2 contributions must cancel

$$\Longleftrightarrow \quad \int d\Omega \, |f|^2 = \frac{-4\pi}{k} \operatorname{Im} f_{\theta=0} .$$

yet another way to arrive at the optical theorem !

World Domination

DoFs

quarks & gluons

N π's

π, σ, ω

A, Z α, β, γ

hadrons + stat. mech.

quarks + stat. mech.

↓

↙ Universe life time

LQCD → Spectroscopy ~ Monte Carlo / Sign problem / Pure Gauge

LQCD → FTQCD

QCD → Schwinger Dyson Eqs. ~ Continuum / Function / Functional

QCD → pQCD ~ asymptotic freedom. / resummation. / NR phy.

χpt EFTs (non)linear σ model

LECs sym.

V_{NN} pot. model Walecka model

Schrödinger eqs. $\pi, \sigma, \omega, \rho, a, \ldots$

scattering theory

Atomic Nucleus Brückner HF GFs.

Liquid Drop Model

Shell Model

Nuclear Matter

HICs Walecka den. functional

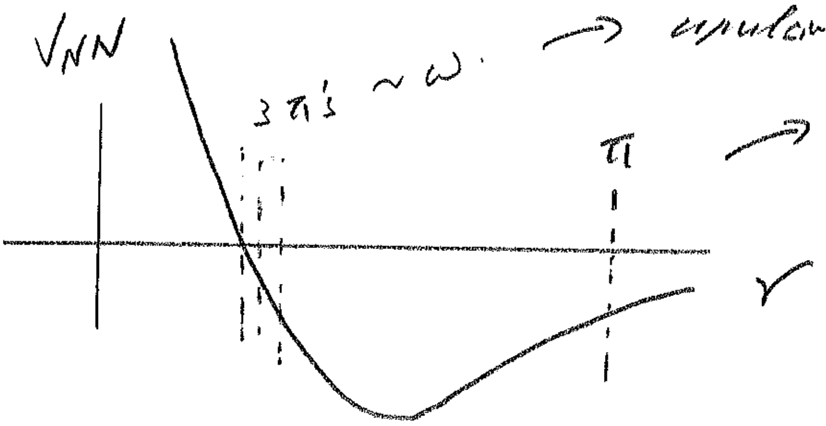
HRG Fermi Gas.

Quark Matter NJL

Functional approach.

Astrophysics. NS PNS EoSs composition.

BHs



→ repulsive

→ attraction

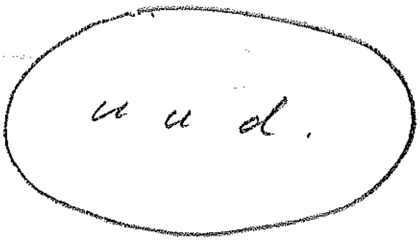
VdW of
QCD
(residual)

how to understand short range repulsion in nuclear force ?

why $\omega \to$ repulsive ?

$\pi \sim \sigma \to$ attraction ?

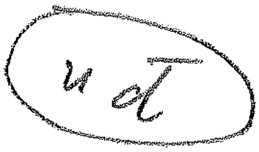
SCSB



$M_p \sim 938$ MeV

m_u
$m_d \sim$ few MeV
(Higgs)

↙ how do we know?



$M_\pi \sim 140$ MeV

what if mass → 0.

$M_p \to$?

$M_\pi \to$?

macroscopic ; phenomenologies

eff. theories : how to build things up ?

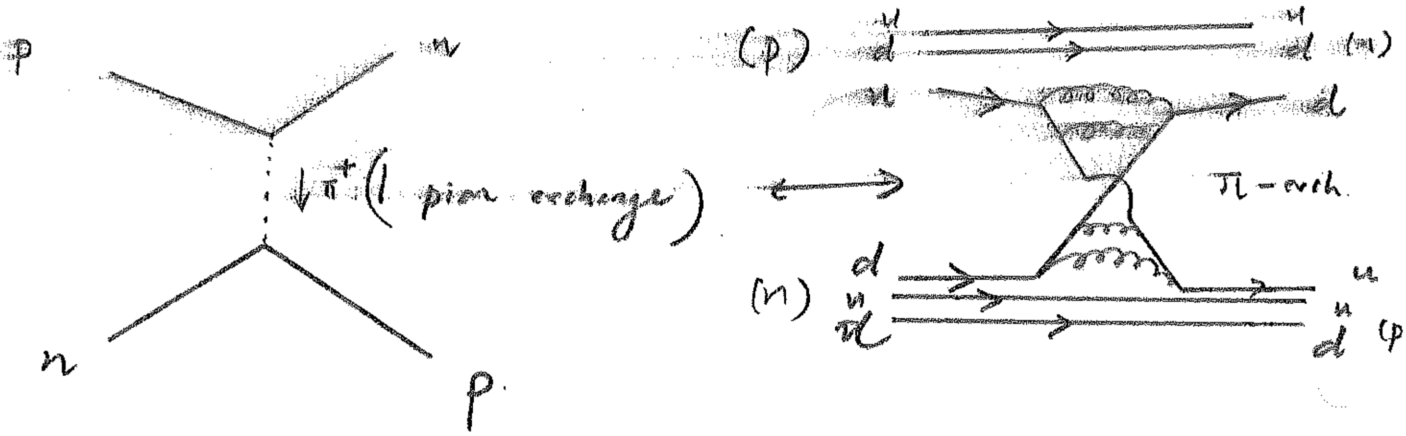
microscopic ; fundamental

QCD

$\sim$ χ_{pt} — π as dof

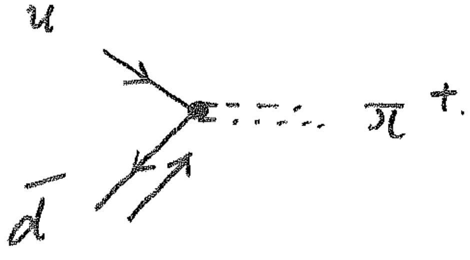
$\sigma \sim \pi\pi$
$\omega \sim \pi\pi\pi$

hadron forces $\sim$ π ; σ, ω exch



how to generate a Bound state is a hard physics problem.

see P. Hoyer.



BSE.

11. 2025

Green's function basics

①

Simple example in Maxwell's eqs.

$$\nabla\cdot\vec{E} = \rho_{\vec{x}}$$

$$\vec{E} = -\nabla\varphi_{\vec{x}}$$ electrostatic potential $\varphi_{\vec{x}}$

$$\Rightarrow -\nabla^2\varphi_{\vec{x}} = \rho_{\vec{x}}$$

→ what do we mean by this?

$$\Rightarrow \varphi_{\vec{x}} = \frac{-1}{\nabla^2_{\vec{x}}}\rho_{\vec{x}}$$

$$= \int d^3x' \underline{\frac{-1}{\nabla^2_{\vec{x}}}\delta^3_{\vec{x}-\vec{x}'}}\rho_{\vec{x}'}$$

GF : $G_{\vec{x}\vec{x}'}$

$G_{\vec{x}\vec{x}'}$ satisfies

$$-\nabla^2_{\vec{x}} G_{\vec{x}\vec{x}'} = \delta^3_{\vec{x}-\vec{x}'}$$

& trivially

$$-\nabla^2_{\vec{x}}\left\{\int \underline{d^3x' G_{\vec{x}\vec{x}'}\rho_{\vec{x}'}}\right\} = \rho_{\vec{x}}$$

$\updownarrow$ is indeed the soln. : $\varphi_{\vec{x}}$

No need to find a soln for each $\rho_{\vec{x}}$

→ search for the solution of $G_{\vec{x}\vec{x}'}$

$$G_{\vec{x}\vec{x}'} = \frac{-1}{\nabla_{\vec{x}}^2}\delta^3_{\vec{x}-\vec{x}'} = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}\,\frac{1}{\vec{k}^2}$$

$$= \frac{1}{4\pi\,|\vec{x}-\vec{x}'|}$$

FT of $\frac{1}{\vec{k}^2}$

FT

$$\frac{1}{\vec{k}^2} \longleftrightarrow \frac{1}{r}$$

~~$\frac{1}{\vec{k}^2+m^2} \longleftrightarrow \frac{1}{r}e^{-mr}$~~

$$\nabla^2_{\vec{x}}\,\frac{1}{|\vec{x}-\vec{x}'|} = -4\pi\,\delta^3_{\vec{x}-\vec{x}'}$$

$$\Longleftrightarrow \quad G_{\vec{x}\vec{x}'} = \frac{-1}{\nabla^2_{\vec{x}}}\delta^3_{\vec{x}-\vec{x}'} = \frac{1}{4\pi\,|\vec{x}-\vec{x}'|}$$

&

$$\varphi_{\vec{x}} = \int d^3x' \,\frac{1}{4\pi\,|\vec{x}-\vec{x}'|}\,\rho_{\vec{x}'}$$

if $\rho_{\vec{x}} = \delta^3_{\vec{x}-\vec{a}}\, Q$

$$\varphi_{\vec{x}} = \frac{Q}{4\pi\,|\vec{x}-\vec{a}|}$$

the Gauss law is realized as

$$\vec{E} = -\nabla \Phi_{\vec{x}} = \frac{Q}{4\pi R^2}\,\hat{R}$$

$$Q = \int d^3x\, \rho_{\vec{x}} = \int d^3x\, Q\, \delta^3_{\vec{x}-\vec{a}}$$

$$= \int d\vec{A}\cdot\vec{E} = \int d^3x\, \nabla\cdot\vec{E}$$

$$= \int d^3x\, \frac{Q}{4\pi}\left[-\nabla^2 \frac{1}{|\vec{x}-\vec{a}|}\right]$$

Similarly

$$(-\nabla_x^2 + m^2)\, G_{\vec{x}\vec{x}'} = \delta^3_{\vec{x}\vec{x}'}$$

$$G_{\vec{x}\vec{x}'} = \frac{-1}{\nabla^2 - m^2}\, \delta^3_{\vec{x}\vec{x}'}$$

$$= \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}\, \frac{1}{\vec{k}^2 + m^2}$$

$$= \frac{1}{4\pi|\vec{x}-\vec{x}'|}\, e^{-m|\vec{x}-\vec{x}'|}$$

//

Going further

$$\left[\partial_t^2 - \nabla_{\vec{x}}^2 + m^2 \right] G_{xx'} = -\delta^4_{x-x'}$$

$$G_{xx'} = \int \frac{d^4p}{(2\pi)^4} e^{-i p \cdot (x-x')} \frac{1}{p^2 - m^2 + i\delta}$$

$$= \frac{-1}{\partial_x^2 + m^2} \delta^4_{x-x'}$$

Introduce the p^0 ⟷ Energy variable

→ interpretation :

$$\frac{1}{p^{0 2} - \vec{p}^2 - m^2} = \frac{1}{p^{0 2} - \mathcal{E}_{\vec{p}}^2}$$

QFT propagator → QFT = QM + special relativity

$p^0 = \mathcal{E}_{\vec{p}}$ is on-shell

Folklore :

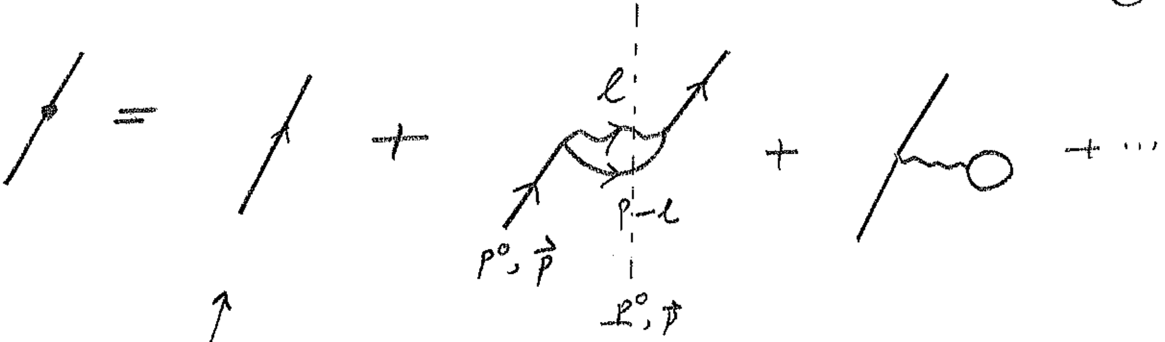
but in QM :

$\Delta t \, \Delta E \sim 1$

$p^0 \neq \mathcal{E}_{\vec{p}} \Leftrightarrow$ off-shell

allows ΔE : borrow E from vac ⇒ Not just dictated by eqn. of motion.

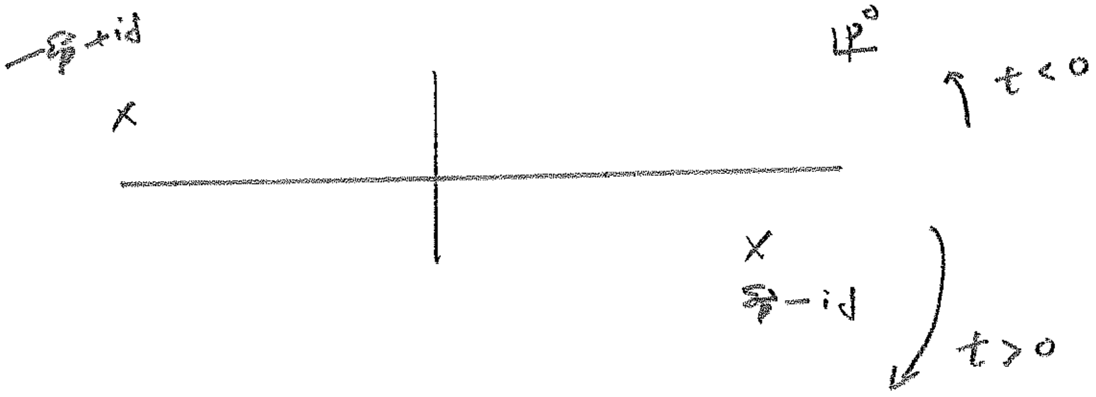
← $\forall$ paths are allowed : Path Integral = quantization



" p^0 is conserved "
but not $= \mathcal{E}_1 + \mathcal{E}_2$
⟷ Not on-shell

$$i G_{p^0, \vec p} = \frac{i}{p^2 - m^2 + i\delta}$$ propagator

$$i G_{t \vec x} = \int \frac{d^4p}{(2\pi)^4} e^{-i p \cdot x} \frac{i}{p^2 - m^2 + i\delta}$$



$$= \int \frac{d^3p}{(2\pi)^3} e^{-i\mathcal{E}_p|t|} e^{i\vec p \cdot \vec x} \frac{1}{2\mathcal{E}_{\vec p}}$$

this is "on-shell"

$$\varphi_x = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec p}}} \left(e^{-i p_{os}\cdot x} a_{\vec p} + e^{i p_{os}\cdot x} a^\dagger_{\vec p} \right)$$

p_{os} is s.t $p^0 = E_{\vec p}$

$$\langle 0| a_{\vec p}\, a^\dagger_{\vec p'} |0\rangle = (2\pi)^3 \delta^3_{\vec p - \vec p'}$$

$\langle 0| T\{\varphi_x \varphi_y\} |0\rangle$

$= \theta(x^0-y^0) \; \langle 0| \varphi_x \varphi_y |0\rangle \; +$

$\quad \theta(y^0-x^0) \; \langle 0| \varphi_y \varphi_x |0\rangle$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\vec p}} \, e^{i\vec p\cdot(\vec x-\vec y)} \, e^{-iE_{\vec p}|x^0-y^0|}$$

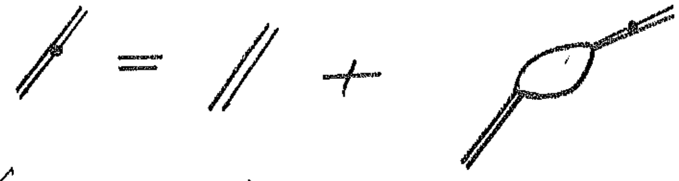
$\longleftrightarrow \quad iG(x-y)$

In fact we can show

$$(\partial_x^2 + m^2) \langle 0| T\{\varphi_x \varphi_y\} |0\rangle = -i\,\delta^4_{x-y}$$

$(\partial_x^2 + m^2)\varphi_x = 0$ .. as eqn of motion.

$$\langle 0| T\{\varphi_x \varphi_y\} |0\rangle \Longleftrightarrow \frac{-i}{\partial_x^2 + m^2}\,\delta^4_{x-y}$$

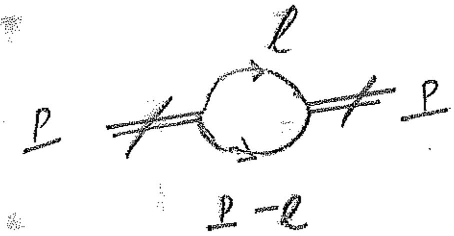


$$iG = iG_0 + iG_0(-i\Sigma)\, iG$$

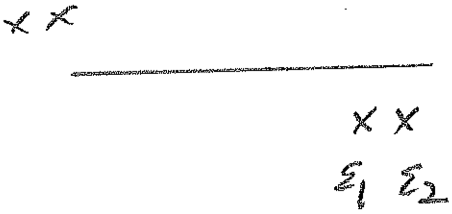
$$\Rightarrow \quad G^{-1} = G_0^{-1} - \Sigma$$

$$-i\Sigma_{(p)} = \int \frac{d^4l}{(2\pi)^4}\ iD_{l_1}\ iD_{l_2}\ (-ig)^2$$

$$\begin{tikzcd} & l_1 = l \quad l_2 = p - l \arrow[dl] \arrow[dr] & \\ \frac{i}{l_1^2 - m^2 + i\delta} & & \frac{i}{l_2^2 - m^2 + i\delta} \end{tikzcd}$$



$$L l_1^0 \ ; \ L l_2^0$$



$\mathcal{C}_2$

$\updownarrow$

if $p^0 > 0$

$$\mathrm{Im}\,\Sigma_{p^0, \vec{p}} = -\frac{1}{2} g^2 \int \frac{d^3l_1}{(2\pi)^3}\frac{d^3l_2}{(2\pi)^3}\frac{1}{2E_1\, 2E_2}\,(2\pi)\,\delta_{p^0 - E_1 - E_2}\,(2\pi)^3\,\delta^3_{\vec{p} - \vec{l}_1 - \vec{l}_2}.$$

$$= -\frac{1}{2} g^2\, \mathcal{C}_2(s).$$

$$G^{-1} = G_0^{-1} - \Sigma .$$

$$\Rightarrow \quad G(p^0, \vec{p}) = \frac{1}{p^{0\,2} - \vec{p}^{\,2} - m^2 - \Sigma_R - i\Sigma_I}$$

CM:

$\vec{p} = 0$

$p^0 \to E$

$$= \frac{1}{E^2 - \bar{m}^2 - i\underline{\Sigma_I(E)}} .$$

$\updownarrow$ eff. mass of the resonance

$$\bar{m}^2 \swarrow \quad m^2 + \Sigma_R(E) .$$

$$i\Sigma_I(E) \searrow \quad + i E \gamma(E)$$

$$\longrightarrow \quad \frac{1}{E^2 - \bar{m}^2 + i E \gamma(E)} .$$

around $E \simeq \bar{m}$

$$\simeq \frac{1}{2\bar{m}} \left(\frac{1}{E - \bar{m} + i \frac{1}{2} \gamma(E)} \right) .$$

$$\swarrow$$

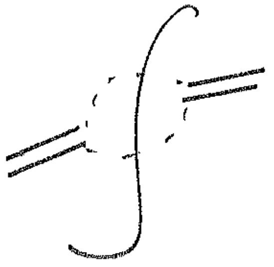
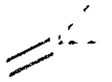
$$\frac{1}{E - m - \Delta(E)}$$

$\Delta_R \leftrightarrow E$ shift

$-2\Delta_I \leftrightarrow$ width.

This is a specific case of general Cutkosky Rule (from unitarity):

$$-2\,\mathrm{Im}\,T = \int d\ell\, |T|^2$$

$-2\,\mathrm{Im}$  $= \int d\ell\ \Big|$  $\Big|^2$

phase space of final states

g^2

$$-2\,\mathrm{Im}\,\Sigma = 2\sqrt{s}\,\gamma(s) = g^2\,\ell_2 .$$

$$\gamma(s) = \frac{1}{2\sqrt{s}} \int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3}\,\frac{1}{4E_1E_2}\,(2\pi)^4\,\delta^4_{P-\underset{os}{p_1}-\underset{os}{p_2}}\,g^2$$

$\updownarrow$ Relativistic Analog of Fermi Golden Rule

$$\gamma(E) = \sum_f |V_{fi}|^2\, 2\pi\, \delta_{E-E_f}$$

E - variable

→ deal w off-shell states also!